Inertial jumps, pressure bursts and acoustic emissions at displacement fluid fronts

Dani Or and Franziska Möbius

Department of Environmental Sciences
Swiss Federal Institute of Technology Zurich (ETHZ)

Cargèse, Corsica 2010
• Passage of imbibition and drainage fluid fronts are common displacement processes in unsaturated porous media (e.g., hydrology - infiltration/redistribution)

• **Advance of a front** - the seemingly smooth macroscopic motion of a front is a result of numerous pore scale rapid jumps pressure bursts challenging the Buckingham-Darcy representation of front motion and energy dissipation

• Discontinuity and indeterminate transport properties and gradients at the front - poorly described flow domain

• **Passage of a front** - pore scale behavior is known to affect resulting hydraulic properties and give rise to hysteresis and dynamic capillary pressure yet often treated qualitatively as “peculiar” processes
Motion of fluid fronts is not “standard” unsaturated flow

Detailed observations of abrupt jumps and pressure bursts

Characterization of velocity distributions, pressure and AE events

Highly inertial motions and various dissipative mechanisms

Modeling pore scale – pressure jumps, displacement and AE
  - The coupled irregular capillaries model
  - Måløy et al.– Invasion percolation, pressure bursts scaling

Displacement fronts – BC and form

Transition length from transient to steady unsaturated flows

Tentative links with pore scale entrapment, hysteresis and dynamic CP
Example - Green-Ampt (1911) infiltration solution

1) A distinct wetting front exists such that the water content behind it ($\theta_0$) remains constant, and abruptly changes to initial water content ($\theta_i$) ahead of the front.

2) The soil in the wetted region has constant properties ($\theta_0$, $K_0$, & $h_0$).

3) Matric potential at the wetting front is constant and equals $h_f$.

4) Combine conservation of mass with Darcy/Buckingham law to obtain infiltration rate:

\[
\Delta \theta \frac{dL_f}{dt} = -K_0 \frac{h_0 - h_f - L_f}{L_f}
\]

\[
\int_0^L \frac{L}{\Delta h + L} \, dL = \frac{K_0}{\Delta \theta} \int_0^t dt
\]

\[
L_f - \Delta h \ln \left[ 1 + \frac{L_f}{\Delta h} \right] = \frac{K_0 \, t}{\Delta \theta}
\]
Example – Richards eq. for horizontal infiltration

\[ \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[ K(h) \frac{\partial h}{\partial x} \right] \]

\[ \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[ D(\theta) \frac{\partial \theta}{\partial x} \right] \]

\[ \lambda = xt^{-1/2} \]

Boltzmann transformation

\[ \frac{\partial \theta}{\partial t} = \frac{d \theta}{d \lambda} \frac{\partial \lambda}{\partial t} = \frac{\lambda}{2t} \frac{d \theta}{d \lambda} \]

\[ \frac{\partial}{\partial x} \left[ D(\theta) \frac{\partial \theta}{\partial x} \right] = \frac{d}{d \lambda} \left[ D(\theta) \frac{d \theta}{d \lambda} \right] \]

\[ -\frac{\lambda}{2} \frac{d \theta}{d \lambda} = \frac{d}{d \lambda} \left[ D(\theta) \frac{d \theta}{d \lambda} \right] \]
Objectives

• To measure morphology and velocity of rapid pore scale interfacial motions during drainage and imbibition in porous media (sintered glass)

• To experimentally link acoustic emissions (AE) and pressure fluctuations with pore scale displacement and interfacial mechanisms

• To model and interpret pressure bursts and AE-producing processes

*Later on.....*

• To link characteristics of pore scale behavior at fluid displacement front with macroscopic transport properties

• To elucidate conditions and characteristic lengths for transition from transient to steady flow regimes behind a front
Observations – pore scale displacement processes

• Imaging with high speed camera (>1000 images/sec) coupled with acoustic emissions (AE) and pressure measurements enable direct monitoring interfacial jumps and characterization of pore scale interfacial motions during drainage and imbibition

• Sintered glass beads with controlled flux or pressure boundary conditions
On acoustic emissions (AE)

- Release of energy due to defect growth, friction between surfaces, or fluid flow
- Energy released propagates as elastic waves
- The proportion of energy depends on localisation of source and duration of event
- Modern acoustic emission (AE) methods are very rapid (sampling in MHz) and sensitive and thus suitable for observing the reach dynamics of many rapid interfacial jumps and reconfigurations taking place during fluid displacement processes in porous media.

Capillary pressure fluctuations

- Rapid measurements show fluctuations superimposed on regular macroscopic capillary pressure changes with drainage.
- These are linked to pore-scale interfacial jumps \( (\text{Haines jumps}) \) with fluctuations characteristics vary with type of \( \text{displacement process; pore space; and front velocity} \). 
- Pressure fluctuations for drainage increase with increasing mean front velocity.

\[
\Delta Z \quad \text{Air entry value}
\]

\[
\text{High flow (100 ml/min; 26 mm/s)}
\]

\[
q = 8 \text{ pores/s}
\quad \text{water}
\quad \text{water + surfactant}
\]

\[
q = 32 \text{ pores/s}
\]
Pressure fluctuations and waiting times

- The width of capillary pressure fluctuations distribution increases with drainage front velocity.
- The waiting times between events become smaller with increased drainage rate.
- In some situations we distinguish less events due to simultaneously occurring bursts.
Macroscopic and pore scale velocity

- Macro- and micro-scale velocities exhibit radically different behaviors
- Motion is composed of numerous local interfacial jumps at significantly higher velocities than mean front speed (some events more than x20 times)
- The motion of these interfaces occurs at high $Re$ with significant inertia
- Interestingly, most of the time a drainage front is pinned and stationary
Local velocity distributions - regular micromodel

- Similar behavior of “mostly stagnant” front is seen for a range of velocities with increasing mean front velocity the distribution of interface jump velocities becomes broader with higher local velocities

- Local Re values are in the range of 100-10000
Bursts and irregular interface jumps - "coil and release"

- The irregular motions of interfaces at a drainage front are attributed to "coil and release" process.
- Pinned interfaces gradually increase curvature (and capillary pressure) until largest throat is abruptly invaded and pressure relaxation ensues.
- A "burst" could be defined as number of pores invaded by one pore on the front before invasion stops and continues elsewhere (Maloy et al. 1992).
- Rapid interfacial release and reconfigurations generate measurable acoustic emissions (AE) and pressure jumps in the fluid.
We seek to directly link individual interfacial displacement in a regularly-spaced medium (4 mm beads) with pinning pressure increase and abrupt relaxation (including AE generation) during drainage.

As mean flow increases multiple pores are invaded simultaneously resulting in complex and cooperative pressure fluctuation patterns.
Most of the time, the main part of the interfacial front is stationary.

Local interfacial jumps have higher velocity than the average front velocity.

High flow rate results in simultaneous emptying of pores along the front.

Pressure fluctuations carry significant information regarding dynamics.
Cooperative pressure jumps – coupled capillaries model

- We solve the momentum balance for interface positions, velocities and fluid pressure for two coupled capillaries under constant withdrawal rate (Q)

$$\frac{2\sigma}{r(h_I)} \cos \theta_d = \frac{8 \mu}{r(h_I)^2} h_I \dot{h}_I + \rho h_I \ddot{h}_I + \rho g h_I$$

$$Q = \dot{h}_{\text{II}} r(h_I)^2 \pi + \dot{h}_{\text{II}} r(h_{\text{II}})^2 \pi$$

Low flow

High flow

Figure 16: Displacement, velocity, and pressure evolution during constant withdrawal of wetting liquid for an exemplary pair of connected columns for low (left) and high (right) flow rate.
Cooperative pressure jumps – *coupled capillaries model*

- Interface positions, velocities and fluid pressure for two coupled capillaries (r=5 mm) under constant withdrawal rate (Q) show displacement and pressure patterns very similar to values measured in regular micromodel.
Cooperative pressure jumps – *coupled capillaries model*

- Minute changes in pore throat sequence (e.g. staggered vs non staggered identical capillaries) result in significantly different displacement and pressure dynamics.

**Non staggered columns**

**Staggered columns**
Måløy, Furuberg, Aker, Hansen – *IP and avalanches*

- Måløy et al. (1992, PRL) and coworkers (Furuberg et al. PRE 1996, Aker et al. 2000) have developed a framework for quantifying the statistical behavior of pressure fluctuations with simplified dynamics using scaling relations based on invasion percolation (and experiments in simple porous media).

- They linked pressure fluctuations with porous media properties, boundary conditions and system size.

- The rate of capillary pressure buildup was quantified based on pore size and BC and the capillary capacitive variable (K) was defined: $K^{-1} = \frac{dP_c}{dV}$ (with $dV = \frac{Q}{n_p}dt$ and $Q$ withdrawal rate [pore/s], $n_p$ number of pores in the front).

- The process may proceed through a series of avalanches of invasion events.

- Pressure burst has a time scale dependent viscosity, mean pressure fluctuation and system size.

$$\frac{dP_c}{dt} = \frac{dV}{dt} \frac{1}{K} = \frac{Q}{(n_pK)}$$
A system size dependency lead to exponential decay of pressure burst with time was derived by Furuberg et al matching the time scale for capillary pressure buildup \( \frac{\text{d}p_c}{\text{d}t} \) with the time it takes to relax a meniscus at distance \( L \) (system size) the characteristic time (containing viscosity \( \eta \) and surface tension \( \sigma \)) \( \tau = \frac{\eta L}{\sigma} \).

The cumulative distributions of pressure jump sizes (b) and time intervals between jumps are shown.

\[ \text{FIG. 2.} \quad \text{(a) Cumulative distribution of intervals between subsequent bursts, } X = T. \text{ Distributions in (a) are vertically shifted by 0.4. (b) Cumulative distribution of pressure jumps, } X = P. \text{ Data from three different experiments are shown as points, simulations as a solid line, and the exponential function as a dot-dashed line. The best experimental fits by the function } C \exp(-\beta X) \text{ are } \beta = 1.27 \text{ in (a) and } \beta = 1.31 \text{ in (b).} \]
Aker et al (EPL 2000, PRE 2000)

- Aker et al. analyzed the distribution of pressure bursts and defined on the sketch as well as waiting times.
- The results of pressure jump size distribution follow power law with exponent close to -2 (considered universal for many self organized critically systems including IP).
- Viscous forces may localize pressure fluctuations along the front ($\Delta P_C \perp$).
- An interesting distinction is made between behavior at the transient front region and a stable region behind it.
Comparison with theory of Måløy et al.

**Graphs:**
- **Equation:** \( y = A \exp(Bx) \)
- **y = C \exp(Dx)\)

**Table:**

<table>
<thead>
<tr>
<th>Particle size (mm)</th>
<th>Mean ( \Delta p ) (mbar)</th>
<th>Mean waiting time ( \Delta t ) (msec)</th>
<th>Pore vol. (mm(^3)) (# pores at front)</th>
<th>Flow rate (pores/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>0.13</td>
<td>220 – 33</td>
<td>1.7 (477)</td>
<td>48 – 1360</td>
</tr>
<tr>
<td>3.1</td>
<td>0.10</td>
<td>564 – 59</td>
<td>11 (115)</td>
<td>8 - 214</td>
</tr>
</tbody>
</table>
Dynamics in short capillaries – inertial regime

- Forces and velocities governing flow in short capillaries (such as rock and soil pores) are vastly different than the familiar equilibrium capillary rise.
- Inertial forces dominate for short capillaries of the order of pore throats.

\[ v \approx \sqrt{\frac{2\sigma \cos(\gamma)}{\rho r}} \]

\[ v = 0.38 \text{ to } 3.8 \text{ m/s} \]

\[ r = 1 \text{ mm to } 10 \mu m \]

Fries and Dreyer (2008)
Inertia and interfacial oscillations in short capillaries

- Inertial motions induce strong menisci oscillations in short pore throats (and in longer large capillaries)
- Rapid oscillations dissipate inertial forces - may generate AE and destabilize interfaces at the front
- Simultaneous events + finite relaxation time $\rightarrow$ lead to complexity and sensitivity to B.C. which may shape phase distribution behind a passing front
Sources of AE during fluid displacement

- Number of AE events and their amplitude increase with decreasing pore size (*power law*); varies with velocity and displacement process (imbibition vs. drainage)
- Various AE-producing interfacial mechanisms are postulated

**Rapid interfacial reconfiguration**

**Snap off processes**

**Pore throat inertial oscillations**

Acoustic emissions - drainage fronts in glass beads of different sizes

Acoustic emissions different drainage velocities

- Number of AE events and their amplitude increase with decreasing pore size (*power law*); varies with velocity and displacement process (imbibition vs. drainage)
- Various AE-producing interfacial mechanisms are postulated

**Rapid interfacial reconfiguration**

**Snap off processes**

**Pore throat inertial oscillations**

Acoustic emissions - drainage fronts in glass beads of different sizes

Acoustic emissions different drainage velocities
Pore scale displacement and unsaturated flow

- Poorly defined conditions at displacement front (hydraulic conductivity and capillary gradient), and inertial liquid motion at high $Re$ deviate from the viscous-laminar Buckingham-Darcy flows

- Davidson et al. (1966) “The size of the pressure increment or redistribution rate will control not only the number of pore sequences which fill and conduct water, but will result in a different water content distribution within the pore sequences”

- Resolving front pore scale dynamics may enable quantification of resulting macroscopic hydraulic properties (e.g., entrapment and hysteresis)
Effects of BC on macroscopic properties

*(left)* Estimates of water diffusivity function for sand under different boundary conditions (pressure step) during drainage (no changes for imbibition!); *(right)* water characteristic curve for various pressure steps drainage and wetting (transient and equilibrium)
Tools for analyses - forces and dimensionless numbers

Key forces:
- Viscous
- Capillary
- Gravity

**Capillary Number** – viscous relative to capillary forces

\[ Ca = \frac{\eta \cdot \bar{v}}{\sigma} \]

**Bond Number** - gravitational relative to capillary

\[ Bo = \frac{\Delta \rho \cdot g \cdot \bar{r}^2}{\sigma} \]

- These dimensionless numbers (force ratios) provide tools for generalization of flow behavior across porous media, fluids, & BC
- Bo --> pore size, contact angle, and surface tension
- *What force is missing here?*
BC → front morphology → phase distribution

- **Meheust et al. (2002)** have shown that when $Bo^*=Bo-Ca$ becomes negative flow becomes unstable (Richards equation no longer applies)

- Experimental images show drainage fronts in same domain for different boundary conditions (gravity and flow rate) – typically associated with small pore to profile scales but may be averaged out at larger scales!

- Lovoll et al (PRE 2004) have shown how the pressure field near finger tip scales with the Ca and this define the resulting invading phase structure
Transition from transient to “steady” conditions

• Lovoll et al (PRE 2004) have shown experimentally and theoretically structural “saturation”/stability occurring a certain distance behind finger tip

• Tallakstad et al. (PRE 2009) demonstrated existence of a characteristic length (and time to steady state) behind displacement front where transition to steady Buckingham-Darcy flow occurs ($t_{ss} \propto Ca^{-0.75}$)

• They proposed transition length depends on pore space and B.C., and marks stabilization of air-water interfaces where standard hydraulic functions become well-defined: $L_{ss} \propto \frac{1}{\sqrt{Ca}}$
Transition from transient to steady conditions

- Similar time or length scales could be derived based on macroscopic arguments of Philip (1969) using Sorptivity (S) and “gravity time” beyond which capillary influence of a front diminishes: \( t \propto \frac{S^2}{K^2} \)

- These understandings may shed new light on interpretation and use of transport functions determined under steady or transient flow conditions.
Summary and conclusions

- Pore scale processes at displacement fronts exhibit complex behavior - pressure jumps and bursts measurable as acoustic emissions
- The highly inertial jumps challenge the standard flow models for displacement fronts and require re-evaluation of the physical picture
- Observational and theoretical tools that consider rapid interfacial motions enable use of information-rich pressure and AE for linking front pore scale behavior with macroscopic hydraulic conditions
- These pore scale processes may shape macroscopic processes ("natural brush") hence hold the key to understanding unresolved macroscopic capillary phenomena such as hysteresis and dynamic capillary pressure
- Transition from intense transient to steady unsaturated flows behind fronts requires different physical representation/interpretation and clarifies limitations of hydraulic functions we routinely use
**Bo* and similar stability criteria**

- Raats [1973] and later Philip [1975] proposed criteria for flow instability - *Flow instability sets in whenever the pressure gradient $G$ at the wetting front opposes the flow*

- Nicholl et al. [1994] provided a similar criterion for fingering in fractured rock: $K_s G - q_{in} > 0$

- The criterion $Bo^*>0$ is more general and equivalent to other stability criteria including Saffman-Taylor criterion $\rho g - \frac{v\eta}{k} > 0$

$$Bo^* = \frac{\text{grav}}{\text{cap}} - \frac{\text{visc}}{\text{cap}} > 0$$

$$Bo^* = \frac{\bar{r}}{\sigma} \cdot W_t \cdot F = \frac{\bar{r}^2}{\sigma} \left( \rho \cdot g - \frac{v \cdot \eta}{k} \right)$$

- Stable displacement (for $Bo^*>0$ or $Bo>Ca$)
- Viscous fingering (for $Bo^*<0$, $Ca>>1$)
Figure 11: Cumulative distribution of burst sizes and time interval can be described with exponential function (left) (Maloy, 1992). The exponents of burst size distribution (red) and time interval (green) are shown versus mean front velocity. Glass beads (■ 1.55-1.85 mm ○ 2.85-3.45 mm) were packed in Hele-Shaw cell.
Effects of boundary conditions on phase entrapment

FIG. 9. Effects of boundary conditions (pressure gradient and drainage front velocity) on macroscopic phase entrapment
Pore scale displacement and unsaturated flow

• The situation at a displacement front is undefined – we have neither hydraulic conductivity nor
• How does this affect unsaturated flow?
  • Steady and unsteady unsaturated flow
  • Transition to steady Buckingham-Darcy flow behind the front – spatial scale for hydraulic functions (velocity interfacial configuration)
• Links to macroscopy hydraulic parameters (doublets, entrapment, dynamic capillary pressure, hysteresis

Fig. 5—Hydraulic characteristics of the aggregated sample.
Outline

• The problem with fluid displacement fronts – jumps, inertia, lack of clear gradients
• Implications for theory, characterization, transport and mixing, post-passage
• Distinguish unsteady vs. steady unsaturated flows – parameters and mechanisms
• What do we see when we zoom in? velocity jumps, pressure bursts
• Why this is expected and how related to properties?
• How to model these processes – mechanistic velocity pressure (small capillary regimes, pair of capillaries)
• Invasion percolation theory predictions
• Acoustic emissions – mechanisms and statistics linked to IP
• Consequences for transport hysteresis, mixing at fronts, colloid transport, etc.
• Why macroscopic theories work after all? Transition zone
• Summary
Bo* and drainage front morphology

- Changes in Bo* (BC, medium) represent different combinations of forces & stability conditions – for drainage, negative Bo* → unstable
- For infiltration - gravity is destabilizing force (opposite sign)

\[
\text{Bo}^* = \frac{\text{grav cap}}{\text{visc cap}}
\]

- Bo* > 0, Bo > Ca → Stable displacement
- Bo* < 0, Ca << 1 → Capillary fingering
- Bo* < 0, Ca >> 1 → Viscous fingering

Meheust et al. "PRE (2002)"
Small scale limitations to Richards equation (drainage)

- **Bo<Ca**
  - Unstable, gravity→ viscous intermittency

- **Bo~0.05**
  - (r ~0.6 mm)

- **Bo>Ca**
  - Stable, capillarity

**Generalized Bo*=Bo-Ca**

- **Ca ≈ Bo sin α - Δθ**

- **Visc. Force** ≈ ηVr >> 1

- **Capillary Force** ≈ σ/σr >> 1
(a) Pressure head evolution during evaporation from saturated sands - pressure ‘jump’ to air-entry value are compared with capillary-saturation curve

(b) Measured air-entry values as function of grain diameter (lines: air-entry pressure based on sphere packing and $R_p=1/3 \, R_G$ rule, respectively)
Interfacial invasion into pores

- Exact path would vary with Ca
Pore-scale observations of colloid transport

- **Imaging with high-speed camera** *(Matahon Ultra, by Videal)*
  (1200 frames/sec; 1024 x 512 resolution)
- **Hele-Shaw cell with sintered glass beads**
  (1-mm diameter, monolayer packing)
- **Polyethylene microspheres** *(Cospheric)*
  (20 – 27 μm, 0.99 – 1.01 g/cm³)

- **Observation variables**
  - **Process**: imbibition and drainage
  - **Flow rate**: 0.42 – 12.5 cm/min
  - **Surface tension**: DI water (73 mN/m); Non-ionic surfactant *(Triton X 100, 1.5x10⁻⁴ mol/L, 32 mN/m)*
Morphology of colloid deposition

- Rapid interfacial jumps and pressure bursts result in different colloid deposition pattern after passage of drainage vs. imbibition fronts.
- In both cases, colloids are retained by straining (in the small spaces between sintered glass beads and wall).
- Behind a drainage front, colloids are retained in residual water ‘rings’.
- Behind an imbibition front, colloids are retained in flow stagnation regions.

Drainage → Imbibition